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Cooperation for volunteering and partially random partnerships

György Szabó and Jeromos Vukov
Research Institute for Technical Physics and Materials Science, P.O. Box 49, H-1525 Budapest, Hungary

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Competition among cooperative, defective, and loner strategies is studied by considering an evolutionary prisoner's dilemma game for different partnerships. In this game each player can adopt one of its coplayer's strategy with a probability depending on the difference of payoffs coming from games with the corresponding coplayers. Our attention is focused on the effects of annealed and quenched randomness in the partnership for fixed number of coplayers. It is shown that only the loners survive if the four coplayers are chosen randomly (mean-field limit). On the contrary, on the square lattice all the three strategies are maintained by the cyclic invasions resulting in a self-organizing spatial pattern. If the fixed partnership is described by a regular small-world structure then a homogeneous oscillation occurs in the population dynamics when the measure of quenched randomness exceeds a threshold value. Similar behavior with higher sensitivity to the randomness is found if temporary partners are substituted for the standard ones with some probability at each step of iteration.

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I. INTRODUCTION

The evolutionary prisoner's dilemma games (PDG) [1–5] are widely applied to study the maintenance of cooperative behavior among selfish individuals under different conditions. This approach provides quantitative predictions about the total income in a fictious society as well as about the measure of exploitation or mutual cooperation when varying the model parameters, e.g., payoffs, set of strategies, topological structure of interaction, rules controlling the choice of a new strategy, noise, and external constraints.

The original PDG describes the interaction between two players whose incomes depend on their choice. In the knowledge of possible incomes the players decide simultaneously whether they cooperate or defect. Henceforth we use rescaled payoff parameters without any loss of generality in the evolutionary PDGs [6]. Thus, for mutual cooperation the players receive unit income providing the highest total payoff for them. Conversely, they receive zero payoff for mutual defection. If they choose different options then the defector reaches the highest individual payoff ($b>1$), which characterizes the temptation to defect; meanwhile, the cooperator receives the lowest one ($c<0$) called sucker's payoff. In the PDG the cooperator's loss exceeds the extra income of defector, i.e., $b+c<2$. These payoffs yield an unresolvable dilemma for the intelligent players because defection brings higher individual income when his coplayer chooses either defection or cooperation. The intelligent players, however, cannot choose defection because it provides only zero income which is the second worst one.

The situation is drastically modified in the evolutionary multiagent systems where the players' income come from iterated PDGs played with several coplayers. For the repeated games the players' decision are determined by their strategy in the knowledge of the coplayers' previous decisions. The evolutionary games allow the players to adopt one of the more successful strategy of the coplayers they interact with. In general, the probability of the strategy adoption depends on the payoff difference. The early simulations performed by Axelrod [5] gave numerical evidence that the presence of the "tit for tat" strategy could sustain the mutual cooperation in systems where all the players interact with each other. The very simple tit for tat strategy cooperates first and afterwards repeats his coplayer's previous decision. Despite its simplicity this is a costly strategy because it requires continuous inspection and the capability to distinguish the coplayers.

The two simplest strategies choose always defection and cooperation, and the corresponding players are called defectors (shortly D) and cooperators (C), respectively. Nowak and May [6] have introduced a two-dimensional cellular automaton where the players are allowed to follow the C or D strategies. Their simulations demonstrated that the cooperators invade the territory of defectors along straight line fronts, while the defector's invasion is preferred along the irregular boundaries. The competition between these invasion processes can maintain the coexistence of D and C strategies with a population ratio depending on the model parameters. Subsequent investigations have demonstrated that the noisy effects make the boundaries irregular giving more chance for defection [7,8]. Furthermore, the cooperation is favored when allowing empty sites on the lattice [7,9]. In this case the empty sites can be considered as "sterile defectors" (for $c=0$) blocking the spreading of defection. After all, the short range interactions between the localized players support the maintenance of cooperation under some conditions (e.g., for small b values) even if the players can follow the simplest (C or D) strategies [6–8,10,11]. Despite the advantage of local interactions the defection prevails in these spatial models if b, the temptation to defect exceeds a threshold value dependent on the evolutionary rules.

In the last few years the investigations of the spatial PDGs are extended to different social networks that takes the present level of transportation and communication into account [12–15]. The extensive research is generally focused on the exploration of those conditions providing the highest total income for such players.

Very recently the introduction of loner (L) strategy is suggested to prevent the uniform defection in the spatial evolutionary PDGs even for large b values [16–18]. The loner strategy represents those players who wish to avoid the risk of exploitation. For this purpose they decline participating in the PDGs and are satisfied by a lower income shared with the coplayer. The coexistence of the three strategies is sustained by the cyclic dominance (L invades D invades C invades L) that yields a proper self-organizing pattern on the spatial evolutionary games. Similar conclusions were already drawn in a previous model where instead of the loner strategy the players could choose the tit for tat strategy under some external constraint favoring random conversion to the cheapest cooperative strategy C [19]. The voluntary PDG with the D, C, and L strategies, however, implies a more convenient background for studying the effect of partially random partnerships.

Henceforth we will study the emerging spatiotemporal patterns of strategy populations as well as the average payoffs affected by different partial randomness in the partnership. More precisely, we study two particular cases with restricting the number of coplayers to be four. In the first case we construct a fixed network by substituting random links for a portion of the nearest-neighbor links of a square lattice. In the second case the standard coplayers are temporarily replaced by random ones with some probability for each step of iteration. We make a comparison between the quenched and the annealed randomness in the partnerships. This approach excludes those effects coming from the fluctuation of the number of neighbors as it happens for diluted lattices [7,9] and different social networks [12–15].

II. VOLUNTARY PRISONER'S DILEMMA GAMES FOR DIFFERENT PARTNERSHIPS

We consider an evolutionary PDG with players who can follow one of the above mentioned D, C, and L strategies. For this set of strategies the payoffs for two players (called X and Y) are tabulated as

$X\backslash Y$	D	C	L
D	$0\backslash 0$	$b\backslash c$	$\sigma\backslash\sigma$
C	$c\backslash b$	$1\backslash 1$	$\sigma\backslash\sigma$
L	$\sigma\backslash\sigma$	$\sigma\backslash\sigma$	$\sigma\backslash\sigma$

where σ $(0<\sigma<1)$ denotes the payoff of both the loner and his coplayer. In the present models each player's income comes from PDGs with his four coplayers. During the evolutionary process the randomly chosen player (X) is allowed to adopt one of the (randomly chosen) coplayer's (Y) strategy with a probability depending on the payoff difference $[M_X-M_Y]$ as

$$W=\frac{1}{1+\exp[(M_X-M_Y)/K]}, \tag{1}$$

where K characterizes the noise [8,17,18]. This choice of W comprises different effects (fluctuations in payoffs, errors in decision, individual trials, etc.). Note that the adoption probability vanishes if $M_X-M_Y \gg K$, and goes to one in the opposite limit. The width of the transient region is comparable to $2K$. Since the work by Nowak and May [6], the analyses are usually performed for $c=0$ that may be a boundary separating different behaviors [20]. To avoid the difficulties caused by the vicinity of such a boundary, now our analysis is restricted to a typical regime of PDG; namely, henceforth we choose $c=-0.1$, $\sigma=0.3$, and $K=0.1$.

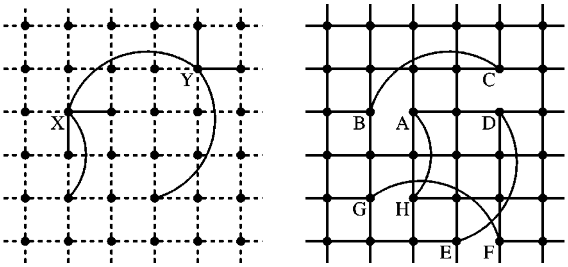

FIG. 1. Temporary coplayers on the square lattice (left) and quenched randomness (right) in the partnerships. On the left-hand side plot the dashed lines indicate the standard links among players located on a square lattice. The solid lines represent the current coplayers of the players *X* and (its coplayer) *Y* if randomly chosen coplayers are substituted for the standard ones with a probability *P*. Short edges pointing outside along the periphery refer to periodic boundary conditions. Partnership with a (quenched) regular small-world structure is shown on the right-hand side plot. The creation of this partnership starts with a square lattice structure where the players are located on the lattice sites and the four nearest-neighbor links define the initial connectivity. First we remove the link between two randomly chosen nearest-neighbor players, e.g., *A* and *B*, and a new coplayer *C* is chosen randomly for the player *B*. Afterwards one of the previous links of *C* is eliminated and a new partner *E* is chosen for the dropped coplayer *D*. This rewiring process is repeated until random links are substituted for the *Q* portion of the original nearest-neighbor links. Finally, the last three-coordinated player (here *H*) is connected to the starting site *A*.

For fixed partnerships the links are defined by the edges of a regular small-world network as explained in Fig. 1. In this case the quenched randomness is characterized by Q, the rewired portion of nearest-neighbor links on the square lattice with periodic boundary conditions. Evidently, this structure reproduces the square lattice in the limit $Q\to 0$. Conversely, for $Q=1$ this structure is equivalent to a random regular graph [21,22] with a uniform degree of four, meanwhile the spatial coordinates become meaningless. Previously we have compared the results obtained for $Q=0$ and 1 [18]. Now our analysis is focused on the small-world structures $(0<Q<1)$ that can well describe the links in social and economical systems [23].

We also study a system where the standard links, defined by the bonds between the nearest-neighboring players staying on the sites of a square lattice, are temporarily replaced with a random one at each step of iteration as explained in Fig. 1. Although this annealed randomness can be introduced in parallel with the above structural randomness for arbitrary Q, our analysis is restricted to those cases when $P=0$ or $Q=0$. Here the number of coplayers is four as above and the exchange probability P measures the strength of annealed randomness. For $P=Q=0$ this system coincides with the traditional one on the square lattice. Evidently, mean-field type behavior is expected if $P=1$ (for arbitrary Q). At low P values, the strategy adoption mechanism prefers the cooperation to defection on the linked (interacting) sites as described for the traditional spatial models [6]. Namely, the defector is punished (receiving reduced income) if one of its coplayer adopts its strategy, meanwhile both the ''ancient and offspring'' cooperators benefit from the strategy adoption [6].

A straightforward consequence of the above strategy adoption mechanism is the appearance of three (homogeneous) absorbing states where the system stays forever after having reached it. For the homogeneous C state the players have the maximal total payoff ($m=4$ per players). This state is unstable against the defector's invasion and finally the defectors dominate the whole territory despite they will receive the lowest average payoff ($m=0$) in the final state. The homogeneous D state is also unstable because the offsprings of a single loner will occupy all the sites sooner or later. Against the loners the cooperators take the advantage of cooperations and they will dominate the system after some transient. Consequently, a rock-scissors-paper cyclic dominance will govern the system evolution in a wide range of parameters [16–18].

It is emphasized that the cyclic dominance is not built directly into the payoff matrix because the loners play a draw with either defectors or cooperators. In fact the cyclic feature of invasion appears at the boundaries separating the homogeneous domains and the formation of domains is supported by the applied strategy adoption mechanism.

In the stationary state the coexistence of the D and C strategies (without L) is possible in a small region of parameters. For example, the stationary frequency of loners vanishes continuously when decreasing the value of b for $c=0$ [18]. This transition belongs to the directed percolation universality class [24,25]. In fact, the survival of loners requires the presence of defectors with a sufficiently large frequency (D feeds L). In the subsequent sections our attention is focused to those situations satisfying this condition.

III. EVOLUTION OF STRATEGY POPULATION

The Monte Carlo (MC) simulations are performed by varying the value of b, Q, or P for fixed c, σ, and K value as mentioned above. In this system one of the homogeneous absorbing states is reached within a short time for small system sizes (due to the fluctuations). To avoid this undesired effect our simulations are carried out on such a large system (consisting of $N\simeq 10^6$ players) where the amplitude of population fluctuation is significantly less than the minimum of the corresponding average value.

The MC simulations are started from a random initial distribution of strategies where the average frequency of each strategy is 1/3. Sometimes we have checked that the stationary states and limit cycles are independent of the initial compositions. During the simulations we have recorded the current frequencies [$\rho_D(t)$, $\rho_C(t)$, and $\rho_L(t)$] and payoffs [$m_D(t)$, $m_C(t)$, and $m_L(t)$] for each strategy.

In this model the sum of the strategy frequencies is fixed [$\rho_C(t)+\rho_D(t)+\rho_L(t)=1$] therefore the time evolution can be illustrated by trajectories on the ternary phase diagram. Figure 2 shows several examples of what can happen when varying only the partnership for fixed model parameters. All these trajectories are decorated by noise with an amplitude comparable to the line thickness for $N=10^6$.

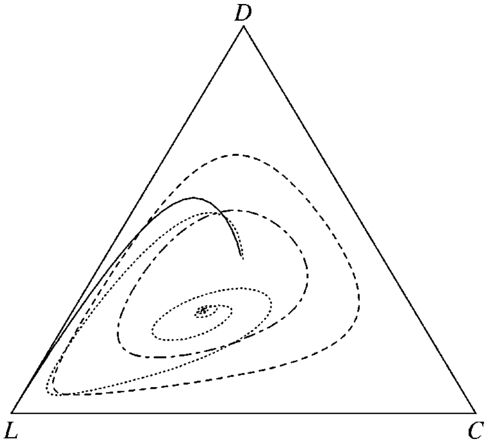

FIG. 2. Trajectories on ternary diagram if the system is started from a random initial state for $b=1.5$. The solid line shows the system where all the coplayers are chosen randomly ($P=1$ or mean-field limit). Evolution on the square lattice ($P=Q=0$) is illustrated by the dotted line. For a weak annealed ($P=0.03$) or quenched ($Q=0.03$) randomness the system tends toward a limit cycle indicated by the dashed and dash-dotted lines.

In agreement with the expectation the MC simulations reproduce the results of the classical mean-field approximation for $P=1$. In this case the system tends towards the uniform loner state independently of the initial state [16,18]. This means that such a system cannot utilize the advantage of cooperation in the final state.

In the pure spatial version ($P=Q=0$) the evolution of strategy frequencies follows a shrinking spiral trajectory towards a fixed point. Around this fixed point the frequencies fluctuate with an amplitude vanishing in the limit $N\to\infty$. Figure 3 shows a typical snapshot on the spatial distribution of strategies. This domain structure evolves very fast because of the motion of invasion fronts, meanwhile its main (statistical) properties remain unchanged in a sufficiently large system. Unfortunately, our knowledge about this type of states is very poor although its geometrical features are already investigated in some simple models having cyclic symmetries [26–28].

This self-organizing spatiotemporal pattern is maintained by the cyclic invasion processes. Disregarding the noisy effects, the D, C, and L strategies follow cyclically each other at each site. These local (cyclic) alternations, however, are not synchronized in the (noisy) spatial models controlled by only short range interactions. Similar patterns can be observed for sufficiently low values of P and Q.

Dramatically different behavior occurs when the fraction of long range interactions (random links) exceeds a threshold value. In this case the system evolves (along expanding or shrinking spiral trajectories) towards a limit cycle indicated in Fig. 2. The visualization of this behavior shows clearly the emergence of global (synchronized) oscillation as demon-

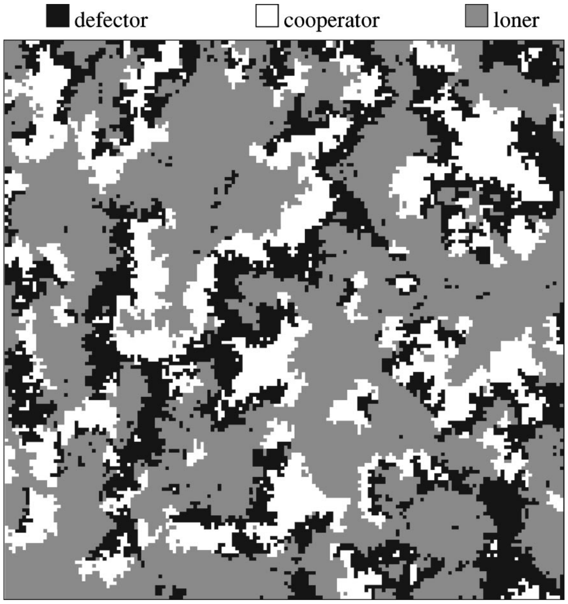

FIG. 3. A typical snapshot on the distribution of the three strategies on a square lattice for $b=1.5$. The different gray scales of strategies are defined at the top.

strated by subsequent snapshots in Fig. 4.

Along the limit cycles the strategy frequencies vary periodically with a typical periodic time $\tau \simeq 50$ MCS, where during the time unit MCS (MC steps per particles) each site has a chance once on average to adopt one of the neighboring strategies. The periodic time increases monotonously with the amplitude.

The emergence of global oscillation (as a Hopf bifurcation) is already well studied in the (mean-field type) Lotka-Volterra models (for a recent survey see the book by Hofbauer and Sigmund [1]). Furthermore, similar transition was reported by Kuperman and Abramson [29] who considered a (cyclic) three-state epidemiological model on the traditional small-world structures suggested by Watts and Strogatz [23].

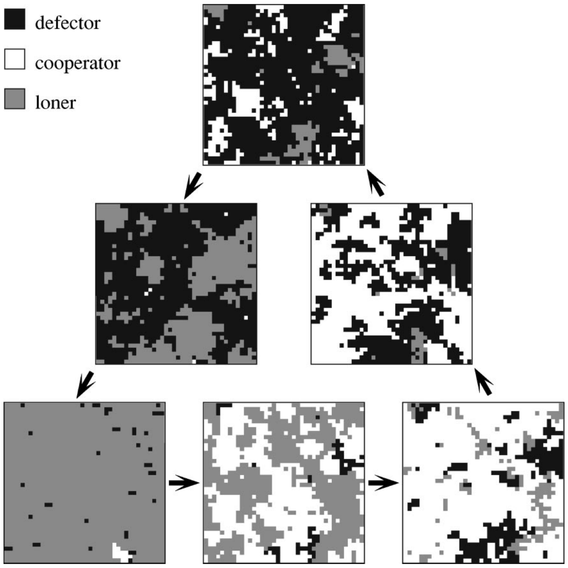

FIG. 4. Typical subsequent patterns occurring along the limit cycles shown in Fig. 2. These snapshots are small parts (40×40 sites) of a larger "homogeneous" phase.

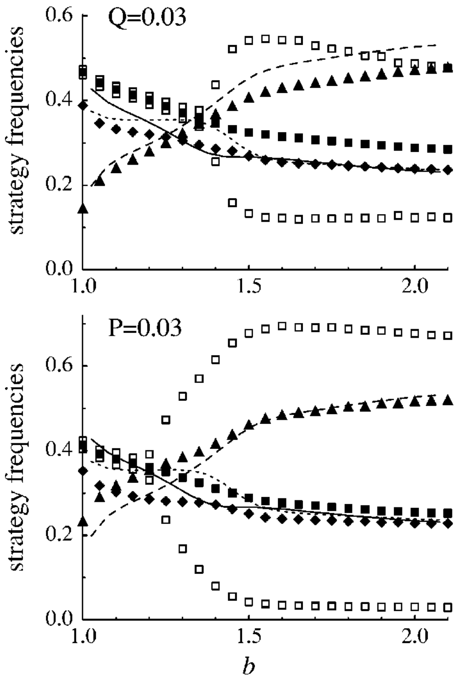

FIG. 5. Monte Carlo data of strategy frequencies as a function of temptation to defect. The solid, dashed, and dotted lines illustrate the frequency of defectors, cooperators, and loners on a square lattice. For quenched (top) and annealed (bottom) randomness in the partnership the closed squares represent the average frequency of defectors and the open squares show their maximal and minimal values due to either fluctuation or homogeneous oscillation. Closed diamonds and triangles indicate the average frequency of cooperators and loners.

In general, the amplitude of global oscillation increases with P (or Q). For some region of parameters, however, the increasing spiral trajectories reach the edges of triangle (see Fig. 2) and the evolution ends in one of the homogeneous state as will be discussed later on.

IV. AVERAGE STRATEGY FREQUENCIES AND PAYOFFS

For quantitative analysis we have determined the average values of strategy frequencies $\overline{\rho_s}$ ($s=D$, C, and L) and payoffs $\overline{m_s}$ by averaging the recorded data over a long sampling time. Besides it we have evaluated the total payoff per players defined as $\overline{m}=\overline{\rho_D}\,\overline{m_D}+\overline{\rho_C}\,\overline{m_C}+\overline{\rho_L}\,\overline{m_L}$. During the global oscillations these quantities are averaged over several hundred cycles. At the same time both the maximum and minimum frequencies of strategies are determined to characterize the amplitude of global oscillation.

First we consider quantitatively the effect of the temptation to defect (b) on the strategy frequencies for three different partnerships. As mentioned above, in the pure spatial model ($P=Q=0$) the three strategies coexist with stationary frequencies. The corresponding MC data are denoted by lines in Fig. 5 where these results are contrasted with those we obtained for a weak quenched ($Q=0.03, P=0$) and annealed ($P=0.03, Q=0$) randomness.

Note that, in contrary to the naiv expectation, the defector frequency decreases when b is increased although such an external manipulation favors the defectors directly. In the traditional rock-scissors-paper games the external support of one of the strategies (or species) initiates a (similar) complex process [30,31]. Namely, during the cyclic invasions the supported strategy will feed its "predator" and decrease the frequency of its "prey" involving a reduced (indirect) attack against the predator. As a result of all these processes the predator of the favored strategy benefits from this external support. In the present case it means that the loner's frequency increases monotonously with b. This tendency remains valid even in the oscillating phases.

Figure 5 shows clearly the striking enhancement of the difference between the maximum and minimum value of $\rho_D(t)$ when $b > b_1$. The threshold values are estimated as $b_1(P=0.03)=1.20(3)$ and $b_1(Q=0.03)=1.38(3)$. In agreement with the expectation, similar behavior can be observed when considering the maximum and minimum values of $\rho_C(t)$ and $\rho_L(t)$ (the illustration is omitted to avoid confusion in Fig. 5). Evidently, such a characterization of the global oscillation involves some uncertainties and size effects. For sufficiently large sizes, however, the maximum (and minimum) frequencies become practically independent of the system size during the global oscillation. Conversely, in the stationary region the maximum and minimum values tend towards the corresponding average values when increasing N.

At first glance (see Fig. 5) the effects of a weak annealed and quenched randomness in the partnership seem to be similar. In both cases the global oscillation occurs as a Hopf bifurcation when b is increased. The comparison of these two systems indicates that $b_1(P) < b_1(Q)$ and the amplitude of global oscillation is larger for the annealed randomness if $P=Q>0$. This means that the variation of the strategy populations is more sensitive to the annealed randomness. Here it is worth mentioning that similar features were described by Zanette [32] who considered a propagation process on both annealed and quenched (nonregular) small-world networks.

To have a better contrast between the two types of randomness now we compare the P and Q dependence of the strategy frequencies and payoffs for the same system parameters, i.e., $b=1.5$, $c=-0.1$, $\sigma=0.3$, and $K=0.1$. For annealed randomness Fig. 6 indicates the absence of global oscillation if $P < P_1 = 0.011(1)$, global oscillation occurs if $P_1 < P < P_2 = 0.021(2)$, and the maximum and minimum values of defector's frequency reach 1 and 0 for $P > P_2$. More precisely, if $P > P_2$ then the trajectories on the ternary diagram (see Fig. 2) reaches the edges of triangle and the evolution ends in one of the homogeneous absorbing states. The ratio of the different terminations depends on the initial state and the model parameters including the system size. It is found that the defectors do not die out first. The defectors will dominate the system if their predators (loners) die out first (due to the fluctuations) when the trajectory goes past the CD edge in the ternary phase diagram (see Fig. 2). Preliminary results on finite systems indicate that the survival probability of loners increases with P and at sufficiently high P values only the loners can survive (see a typical mean-field result in Fig. 2). Evidently, the threshold values (P_1 and P_2) depend on the model parameters.

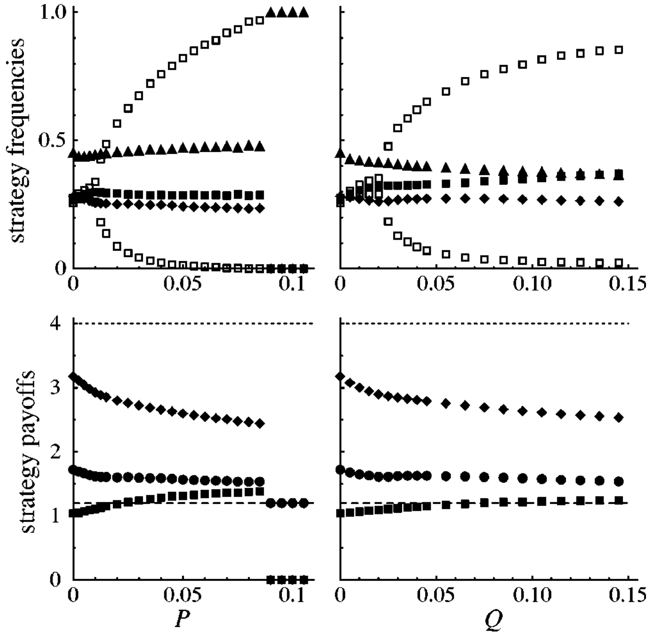

FIG. 6. Effect of random partnership on the strategy frequencies (upper plots) and average payoffs (lower plots). The MC results are plotted as a function of P (left) and Q (right) for $b=1.5$. The symbols as shown in Fig. 5, while the closed circles shows the total payoff per players. The dotted line indicates the maximum value of payoff for mutual cooperation, while the dashed line shows the loner's fixed payoff.

On the other hand, when increasing the measure of quenched randomness (Q) the global oscillation appears at $Q > Q_1 \simeq 0.020(2)$ and it remains stable even on the random regular graph (limit $Q \to 1$). For higher b values, however, the evolution can terminate at one of the absorbing states as described above if $Q > Q_2(b,c,\sigma,K)$.

The rigorous numerical analysis of the second transitions is prevented by the finite-size effects and the slow relaxation towards the limit cycle. Another difficulty comes from the absence of a well-defined order parameter characterizing the limit cycle. We think that further (time-consuming) simulations are required to clarify the main features of these transitions as well as the derivation of phase diagrams which show how the threshold values depend on the model parameters.

Previously we have demonstrated that this system on the random regular graph can be very well described by a simple pair approximation [18]. This calculation shows that the minimum value of ρ_D (along the limit cycle) vanishes linearly with b [18]. An extended version of this technique confirms that the introduction of an additional annealed randomness ($P>0$ for $Q=1$) reduces the threshold value(s) and shifts the evolution towards the mean-field type behavior.

The highest average payoff is always received by the cooperators despite their territories are invaded by the exploiting defectors. One step behind the invasion front, however, the defector's income drops to zero decreasing their average payoff. In the present cases the average defector's income becomes comparable to the loner's fixed income ($m_L = 4\sigma$). The average strategy payoffs are weakly affected by the value of P or Q (as well as by b). This weak effect can be considered as a consequence of the mentioned compensation mechanism of the cyclic invasions. The quantitative analysis indicates the loss of cooperators (i.e., m_C decreases monotonously) when increasing either P or Q. At the same time the defectors benefit in both cases. The increase of defector's income, however, is less than the loss of cooperators. Consequently, the total income decreases if random links are applied at the expense of local links.

Evidently, the strategy payoffs oscillate together with the frequencies during the global oscillations. The increasing amplitude of the global oscillation can cause dramatic consequences only if the evolution terminates in one of the homogeneous absorbing states. Plots on the left-hand side of Fig. 6 exemplify that the loners will prevail the system if $P > P_2 = 0.086(3)$. In this situation the total payoff per players fails to the loner's income m_L.

In comparison to the square lattice the quenched small-world structure enhances the surface (as well as the irregularity) of the domains containing the same strategy inside. This variation in the geometrical features modifies the invasion rates. For example, the enhanced irregularity provides an extra support for the defectors against the cooperators at the corresponding boundaries as mentioned above. Apparently this effect is similar to the increase of b. However, the topological variations will affect the invasion rates for other boundaries separating C and L, or D and L domains. As a result of all these processes, the average frequency and income of defectors increase monotonously with Q (see Fig. 6).

For quenched partnerships ($P=0$) the strategy adoptions take place only at the boundaries separating two domains. For the annealed randomness ($P>0$), however, the new strategy can occur inside the homogeneous domains via a temporary link from arbitrary distance. This possibility can be utilized by defectors (loners) when invading the domains of cooperators (defectors), meanwhile the spreading of a single cooperator inside a loner domain is not supported by the payoff differences. Furthermore, the mutual punishment between the parent and offspring defectors vanishes if the adoption is mediated by a temporary link. At low P values, there is an additional event that provides an extra income for those defectors residing inside their domain. Namely, via the temporary links the cooperators may be easily substituted for those defectors surrounded only by defectors and this solitary cooperator will feed the neighboring defectors until its death. These events together yield that the average defector income increases faster with P than with Q. In the multiagent PDG the increase of $\overline{m}_D$ is always accompanied by a higher loss in $\overline{m}_C$ and $\overline{m}$ (compare the lower plots in Fig. 6).

V. SUMMARY

This work is devoted to study the effect of partially random partnership on the measure of cooperation in a society where the pairwise interaction between the members is approximated by an evolutionary PDG. It is found that the appearance of cyclic invasion between the domains of the available strategies plays a crucial role in the maintenance of cooperation at some level in the evolutionary PDG. In the voluntary PDG strikingly different behaviors occur when comparing the spatial model with those satisfying the conditions of a mean-field approximation. The mean-field model predicts that only loners will survive after a transient process. Conversely, on the square lattice the coexistence of the three strategies (D, C, and L) is maintained by the cyclic invasions resulting in a self-organizing pattern.

We have studied the crossover between these two particular behaviors. For this purpose two types of partially random partnerships are investigated systematically. In the first case temporary partners (from arbitrary distance) are substituted (with a probability P) for the standard ones defined by the nearest neighbors on the square lattice. Tuning the value of P this structure of partnership provides a continuous variation from the spatial evolutionary PGD to the suitable "mean-field model." This system exhibits two transition points if P is increased. Below the first threshold value ($P < P_1$) the self-organizing pattern is disturbed only slightly by the rare temporary links. Within an intermediate region ($P_1 < P < P_2$) global oscillation occurs whose amplitude increases with P and reaches the saturation value at P_2. Above the second threshold value this system tends towards a homogeneous state providing loner dominance for large P.

Similar scenario is found in the second case when the quenched random partnership is created before starting the evolutionary process. In this case the structure of partially random partnership is constructed on the analogy of small-world network [23] conserving the degree of four for each site. The quenched partnership is able to sustain some advantages of the local interaction as discussed above. As a result, the effect of this type of randomness is reduced in comparison to those of annealed randomness. This means that the transitions between the mentioned phases appear at higher measure of randomness. Accordingly, the second transition can disappear in some region of parameters (see an example in Fig. 6). In such situations the prevalence of loner strategy is prevented even for the limit $Q \to 1$.

From the point of view of sociology and economy, our investigations give some evidence that the exploiting strategy (defector) benefits from the increase of randomness in the partnership for the voluntary PDG. Collaterally, the total income decreases due to the enhanced loss of cooperators. The undesired effects are moderated significantly by the introduction of loner strategy (and by the mechanism of cyclic invasions). The advantage of "local interactions" in the maintenance of cooperation is destroyed more efficiently by the annealed partnership rather than by the quenched structure. The simulations indicate that the efforts for cooperation and exploitation (defection) disappear after an increasing global oscillation in the strategy frequencies if the measure of annealed randomness exceeds a surprisingly low threshold value depending on payoffs, noise, etc. Figure 6 shows an example where the stationary mean-field behavior is reached (only loners survive) if 10% of local neighbors are replaced by random temporary partners.

The above investigations raise many questions, demand further systematic researches, and encourage us to look for different mechanisms and conditions that can improve the cooperation in such types of communities. On the other hand, the deeper insight into these phenomena requires different extensions of the methods developed originally for the nonequilibrium statistical physics.

ACKNOWLEDGMENTS

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